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Simple optical feedback loop: Excitation waves and their mirror image

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The influence of feedback mechanisms on the spiral wave dynamics is analyzed both experimentally and numerically. The image of a spiral wave is projected (by means of mirrors) on the photosensitive Belousov-Zhabotinsky reaction in such a way that the spiral wave is forced to interact with its image. Larger reflected light intensity means partial inhibition of the reaction and causes a spiral drift away from its ever-accompanying mirror image. From the point of view of optics, this is a very simple feedback loop, and may open a wide area of research, in view of the possibilities of varying geometry and introducing inhomogeneities. [S1063-651X(97)50101-2]

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Waves in excitable media are prototypes of interdisciplinarity. Indeed, they behave analogously in macroscopic scales in systems that are microscopically quite different: chemical reactions [1–5], flames [6], self-organizing organisms [7], cell surfaces [8], cell interior [9], nervous tissues [10], heart muscle [11], and catalyst surfaces [12]. Furthermore, their computational capabilities make them suitable for parallel image processing [13–15], logic gate networks [16], and pathfinding algorithms [17]. For external control, waves have been manipulated using electric fields [18,19], light gradients [20], or mechanical action [21]. We present a most simple way of manipulation: light sensitive waves on a mirror are driven by their ever-accompanying mirror image.

Common to all excitable media [22–24] is that they can support waves that retain their shape and amplitude by extracting energy from the medium. These waves have an excited front running into the medium by virtue of diffusion-triggered autocatalysis, leaving behind a nonexcitable region. In an homogeneous medium, mostly circular and spiral waves are obtained.

The medium investigated in this work is the Belousov-Zhabotinsky (BZ) reaction, catalyzed by the ruthenium bipyridyl complex $\mathrm{Ru(bpy)}_3^{2+}$ (see Refs. [14] and [25]). Light induces the release of Br^-, which decreases the velocity of propagation. To avoid convection the catalyst was immobilized in a silica-gel matrix in a Petri dish (gel thickness: 0.9 mm; preparation as in Ref. [26]). The solution (0.18 M NaBr, 0.33 M malonic acid, 0.39 M $\mathrm{NaBrO_3}$, and 0.69 $\mathrm{MH_2SO_4}$; volume equal to that of the gel) was poured onto the gel (temperature $=25\pm1$ °C). White light, leaving horizontally from a 250-W halogen lamp, passed first a diffusive screen, and then was reflected by an obliquely placed mirror. Then, the light reached the reagent, which was placed hoizontally on a plane silver mirror. (Glass thickness between reflecting surface and lower surface of the gel: 1.5 mm. In the absence of the silver mirror below, the light intensity on the reagent surface was equal to 33 $\mathrm{W/m^2}$ and was homogeneous within 10%. These deviations from homogeneity, however, were not relevant since (i) imposing a gradient twice as large did not change observations, and (ii) a change in the spiral chirality at the same illumination yielded axially symmetric dynamics. Recording was done with a vertical video camera via an interference filter (450.6 nm). Mirror shifts d (horizontal distances between the chemical wave and its image, as seen from the viewpoint of the camera) were adjusted by varying the orientation of the oblique mirror.

For calculations we used the Oregonator model, modified to include light-induced bromide production [27]:

$$\frac{\partial u}{\partial t}=\frac{1}{\varepsilon}\left(u-u^2-(fv+\phi)\frac{u-q}{u+q}\right)+\Delta u,\quad \frac{\partial v}{\partial t}=u-v.$$

u and v describe $\mathrm{HBrO_2}$ and catalyst concentrations, and ϕ represents the light-induced flow of Br^-. We set $\varepsilon=0.02$, $q=0.002$, and $f=3$. The mirror image was roughly approximated as follows: the regions in which $u>0.3\,u_{\max}$ ($u_{\max}$: maximum of u) were all shifted a distance d; within these shifted regions we set $\phi=0.03$ (value optimized for best agreement with measurements) and $\phi=0$ elsewhere.

Figures 1(a)–1(d) [respectively Figs. 1(e) and 1(f)] exemplify the observed (respectively simulated) spiral drift. A more quantitative account of the observations is given in Fig. 2. For statistical assurance, 40 experiments were performed (five shift values times eight repetitions) of approximately 2 h each, as summarized in Fig. 3(a). Figure 3(b) shows the corresponding results from simulations. Clearly, there is a drift velocity maximum for $0<d<\lambda$, and qualitative agreement between measurements and calculations. In contrast to the drift observed by parametric resonance [19], the present results do not depend on the initial spiral tip phase. The case $d=0$ could not be measured directly because of difficulties in exact alignment of wave and image, but extrapolation in Fig. 3(a) seems to lead to the origin, thus indicating zero drift at $d=0$ (as observed numerically). At a shift lower than one wavelength, we obtain a drift velocity perpendicular to the shift $v_\perp$ in the opposite direction in calculations. This, again, could not be confirmed in experiments because of excessive

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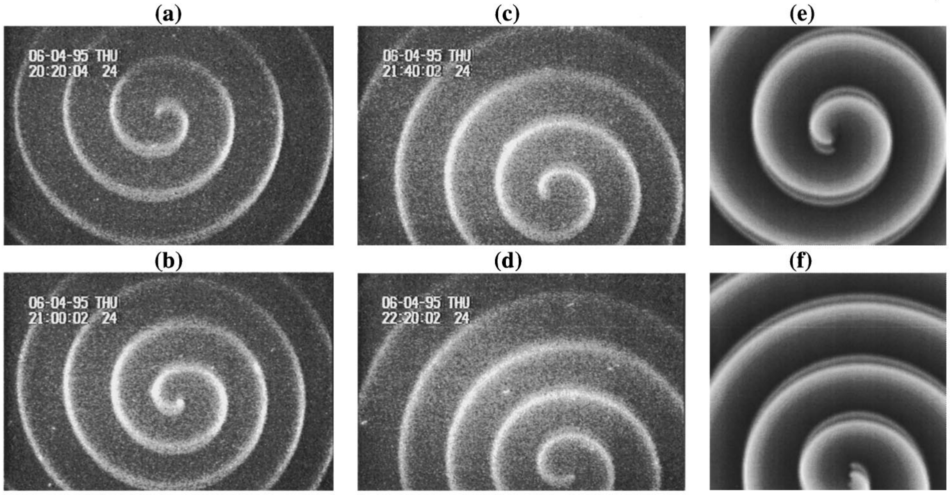

FIG. 1. (a)–(d): Experimentally observed drift of a spiral wave under the action of its mirror image. The image lies upwards and is faintly visible as a broadening of the lower and upper arcs of the spiral: wavelength; 2.2 mm; period 45 s; shift of the mirror image; 0.4 mm; times: 0 (a), 40 min (b), 80 min. (c), 120 min. (d). (e), (f): Calculations using the modified Oregonator model: times 0 (e) and 150 time units (f): spiral period, two time units; wavelength, ten spatial units; image shift two spatial units.

variance. The sign of $v_\perp$ depends on the spiral chirality; all results presented in this work are for counterclockwise rotation.

Inspection of the behavior of the tip, both in experiments and in calculations, renders the following explanation for the drift. For the present conditions and without a mirror, the tip moves circularly around the spiral core. Suppose now the image shift lies towards the north of the wave and that the spiral rotates counterclockwise. Note that the highest reflected light intensity (highest inhibition) corresponds to the image of the excited wave front because this front acts as a ‘‘window’’ between incident light and horizontal mirror. Note also that the image of the core is homogeneous and darker than the image of the front wave. As the tip passes through the north or south, the image shift is approximately zero, having no significant effect. Also, there is no effect as the tip passes the west, since the inhibitory image lies on the waveback. There is only an effect around the eastern position of the tip, where the velocity normal to the front decreases as the wave runs into its image, while the growing velocity of the tip remains unaffected. Consequently, the core, and thus, the whole spiral moves mainly towards the south. This mechanism is the opposite of phototaxis and thus different from the phenomenon observed in a light gradient [20]; there, drift occurs because the core radius is larger (respectively smaller) as the tip moves to the lighter (respectively darker) regions.

The image-reagent pair described here is an optical feedback loop in which the camera-monitor system is only used for recording, and is not (as in Ref. [28]) a necessary optical amplifier within the loop. Optical loops have been proposed for image processing and optical computing [29,30]. The simple loop presented here may be broadly extended by us-

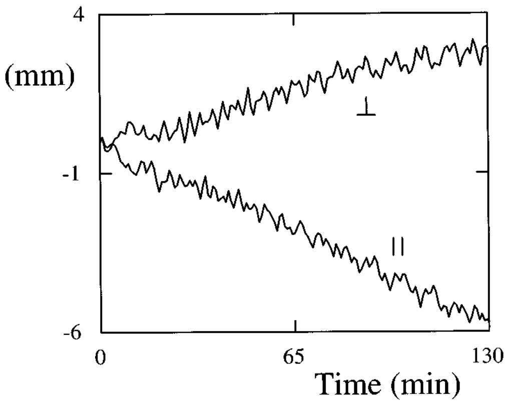

FIG. 2. Position of the spiral tip during the measured drift, relative to the initial tip position. These positions were obtained by mouse-clicking on stored images. The position components perpendicular (⊥) and parallel (∥) to the mirror image shift are depicted. The drift velocities $V_\perp$ and $V_\parallel$ in Fig. 3 are obtained by fitting a straight line to the positions of the spiral tip shown here.

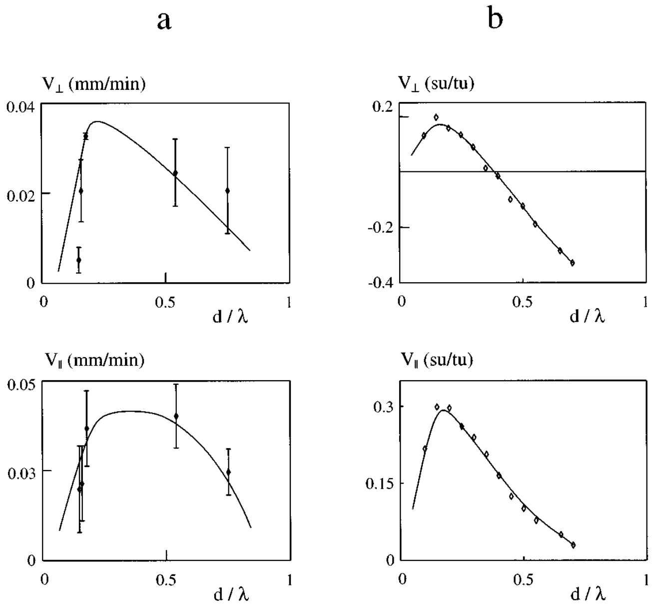

FIG. 3. Ordinates: measured drift velocity components perpendicular $V_\perp$ (upper) and parallel $V_\parallel$ (lower) to the mirror image shift; abscissa: image shift d relative to the wavelength λ; error bars: standard deviation obtained from eight repetitions of each experiment (one of them being exemplified in Fig. 2). (b) Drift simulations [in spatial units (su) and time units (tu)].

ing inhomogeneous, tilted or curved mirrors, or by intercepting other optically active materials.

This simple mechanism can be used for understanding the behaviors observed in more complex systems where feedback mechanisms take place. An example is the cardiac tissue where formally the same kind of waves propagate. Here the propagation of an autowave induces a change in the elastic properties of the medium. On the other hand, the elastic properties of the medium influence the propagation of autowaves [31], This means that there exists a feedback process that may have some influence on typical cardiac processes such as arrhythmias.

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